INESTERA: Journal of Interdisciplinary Research
Vol. 1, 2 (April-June, 2026), pp. 79-90
EISSN: 3124-8586, DOI: https://doi.org/10.59525/inestera.1573

Comparing Integer and Fraction Multiplication Problem-Solving Across Multiplicative Structures Among Rural Sixth-Grade Students in Taiwan

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ARTICLE INFO

Keywords:

Multiplicative Reasoning;
Fractions;
Multiplicative Structures;
Rural Students.

Article history:

Received 2026-04-13
Revised 2026-06-19
Accepted 2026-07-28

ABSTRACT

Multiplication is fundamental in primary mathematics, yet students often struggle when extending their understanding from integers to fractions. This study investigated the multiplication problem-solving performance of sixth-grade students in rural primary schools in Taiwan, focusing on differences across four multiplicative structures: equal groups, multiplicative comparison, rectangular area, and Cartesian product. A researcher-developed paper-and-pencil test was administered to a total of 178 sixth-grade students (87 boys and 91 girls) from 15 rural schools, and the data were analysed quantitatively. The results show that students consistently performed better in integer multiplication than in fraction multiplication across all structures. Equal groups problems yielded the highest accuracy rates, whereas rectangular area and Cartesian product problems showed lower performance. In addition, the performance gap between integer and fraction contexts varied by structure, ranging from 10.8% in equal groups problems to 23.4% in rectangular area problems. These findings suggest that different multiplicative structures may impose varying cognitive demands on students, which may contribute to differences in fraction multiplication performance. By highlighting structure-based variations in performance, this study provides a more nuanced understanding of students' multiplicative reasoning and raises questions about whether instruction that explicitly addresses structural differences could support student learning, particularly in rural educational contexts.

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1. INTRODUCTION

Mathematics plays a fundamental role in human knowledge and is essential across scientific and everyday contexts. In today's information-rich society, mathematical applications are ubiquitous, underscoring both their importance and their central role in basic education. In everyday life, multiplication of whole numbers is one of the most frequently used operations and is considered a fundamental component of early mathematical learning (Bell et al., 1989).

Fractions occupy a central position in mathematics education; however, they are also widely recognised as one of the most challenging topics for students. Although students may acquire procedural skills in fraction operations, they often lack a deep conceptual understanding of the underlying relationships. This gap between procedural fluency and conceptual understanding has been consistently identified as a key issue in fraction learning (Cramer et al., 2002). Moreover, students' understanding of fractions is closely related to their later mathematical development, particularly in areas such as algebra and proportional reasoning.

Compared with multiplication of whole numbers, fraction multiplication requires more sophisticated reasoning. Rather than relying on repeated addition, students must coordinate units, interpret quantities, and understand multiplicative relationships involving scaling and part–whole structures (Empson & Levi, 2011). Research has shown that students often rely on whole-number reasoning when solving fraction problems, which may lead to systematic errors when such strategies are inappropriately generalised. In this regard, multiplicative reasoning is not simply procedural but involves the construction and coordination of composite units (Zwanch & Wilkins, 2021). Recent research has further distinguished different conceptions of multiplicative reasoning, suggesting that students may interpret both correct and incorrect solutions based on their underlying conceptual structures (Ding et al., 2022).

Recent studies have consistently shown that students' performance in fraction-related tasks is influenced by conceptual understanding, numerical representation, and instructional support, highlighting the complexity of fraction learning across different contexts (Braithwaite & Siegler, 2021; Fuchs et al., 2013; Jordan et al., 2024; Liu & Braithwaite, 2023). These findings suggest that fraction learning is not only a matter of procedural proficiency but also involves the integration of multiple cognitive components, which may lead to variations in students' problem-solving performance across different contexts and task structures.

More recent research has also explored the broader cognitive and instructional factors associated with fraction learning. For instance, Xu et al. (2024) suggested that fraction learning involves a complex interaction of conceptual understanding, misconceptions, and learners' cognitive characteristics. In a longitudinal perspective, D'Erchie et al. (2026) found that early algebra knowledge predicts later fraction understanding, indicating that fraction learning is closely connected to broader mathematical development. Additionally, intervention studies have shown that targeted instructional approaches can support students' learning, particularly for those with learning difficulties (Björkhammer et al., 2025).

Although fraction learning is often regarded as challenging, the difficulty of fraction operations may vary across mathematical contexts and task types. Therefore, the present study focuses not only on fraction multiplication itself but also on how different multiplicative structures may influence students' performance in fraction multiplication situations.

Although previous studies have highlighted the distinction between conceptual and procedural understanding in fraction learning, students' conceptual understanding may not be expressed uniformly across different problem contexts. Multiplicative structures provide one way of examining how the same mathematical operation may involve different quantitative relationships and levels of conceptual complexity. Therefore, students' performance across multiplicative structures may reveal how conceptual understanding of fraction multiplication varies according to the underlying structure of a problem.

Multiplicative structures provide an important theoretical framework for analysing students' problem-solving performance. Greer (1992) identified four types of multiplication problems: equal groups, multiplicative comparison, Cartesian product, and area. Although Greer (1992) discussed additional multiplicative situations such as rate and scaling, the present study focused on equal groups, multiplicative comparison, rectangular area, and Cartesian product because these structures are most commonly represented in Taiwan's elementary mathematics curriculum and are frequently encountered in primary school multiplication word problems. Focusing on these four structures therefore allows a closer alignment between the theoretical framework and the curricular experiences of Taiwanese sixth-grade students.

These structures differ in their underlying quantitative relationships and cognitive demands. In particular, problem types involving area and Cartesian product often require more complex reasoning and may pose greater challenges for students.

Based on previous research, different multiplicative structures may be expected to vary in their level of difficulty when students solve fraction multiplication problems. Equal groups problems may show relatively smaller differences between integer and fraction contexts because they are closely associated with repeated addition and familiar multiplication situations. In contrast, area problems may pose greater challenges because they require students to coordinate two continuous quantities and interpret multiplicative relationships within a spatial context. These differences suggest that students' performance may vary across multiplicative structures when comparing integer and fraction multiplication contexts.

From a curricular perspective, multiplication plays a critical role in primary mathematics education. According to the Grade 1–9 Curriculum Guidelines issued by the Ministry of Education in Taiwan (2008), multiplication is emphasised across multiple stages of learning. The 12-Year Basic Education Curriculum Guidelines in Taiwan (Ministry of Education, 2018) further highlight the importance of connecting multiplication concepts to real-life contexts and developing students' problem-solving abilities based on quantitative relationships. These policy documents underscore the foundational importance of multiplication within the Taiwanese educational context and support the need to investigate students' understanding of multiplication in both integer and fraction contexts.

Despite the growing body of research on fraction learning and multiplicative reasoning, relatively few studies have systematically examined how students' performance varies across different multiplicative structures when comparing integer and fraction multiplication contexts. In particular, prior research has tended to focus on overall performance or specific types of tasks, rather than comparing multiple structures within a unified framework. Furthermore, such investigations remain limited in rural educational contexts, which have received comparatively less attention in research on fraction multiplication and multiplicative structures.

Rural schools, also referred to as remote-area schools in Taiwan, are typically located in areas characterised by geographical remoteness, limited transportation access, inadequate living facilities, underdeveloped digital learning environments, or low population density. In such contexts, schools may face challenges related to instructional resources, teacher recruitment and retention, and access to diverse learning opportunities. Consequently, rural education has remained an important policy concern in Taiwan, with continuing efforts to reduce educational disparities and promote equitable learning opportunities. However, the purpose of the present study is not to compare rural students with urban or national populations. Rather, this study focuses on rural sixth-grade students as an underrepresented group in research on fraction multiplication and multiplicative structures, and aims to describe their multiplication problem-solving performance across integer and fraction contexts. In this context, area and Cartesian product problems may warrant particular attention because they often require visual representations, multiple forms of reasoning, and sustained conceptual support.

Given these contextual challenges, it is important to examine how students in rural areas develop their mathematical understanding, particularly when encountering more complex concepts such as fraction multiplication.

Based on these considerations, this study aims to compare sixth-grade students' multiplication problem-solving performance in integer and fraction contexts across different multiplicative structures. Specifically, this study addresses the following research objectives: (1) to compare students' problem-solving performance in integer and fraction multiplication contexts, and (2) to examine how performance varies across four multiplicative structures: equal groups, multiplicative comparison, area, and Cartesian product.

2. METHODS

This study adopted a quantitative research design to investigate the multiplication problem-solving performance of sixth-grade students in rural primary schools in Taiwan. A total of 178 sixth-grade students from 15 rural schools participated in the formal assessment, including 87 boys and 91 girls. All participants provided valid responses and were included in the data analysis. Before the formal administration, a pilot test was conducted with 28 sixth-grade students (15 boys and 13 girls) from two rural schools in Taiwan to establish the reliability and validity of the research instrument.

Before the implementation of the study, the researcher conducted a literature review focusing on multiplicative structures, the role of multiplication in primary mathematics curricula, and fraction learning. Based on these theoretical foundations, as well as existing standardised test items used in Taiwan, the researcher developed a multiplication test aligned with current curriculum content and conceptual frameworks. The test items were categorised into four types of multiplication problems: equal groups, multiplicative comparison, area, and Cartesian product. In this study, "fraction multiplication" refers to multiplication word problems that involve at least one fractional quantity, including whole-number by fraction, fraction by whole-number, and fraction by fraction situations. After the pilot testing, the instrument was revised and finalised for the formal paper-and-pencil assessment. Following data collection, all responses were organised, coded, and analysed.

The research instrument was a researcher-developed multiplication test designed to assess students' problem-solving performance across different multiplication structures. The initial pilot version consisted of 24 items, with equal distribution across the four problem types and numerical contexts (integers and fractions), as shown in Table 1.

Table 1. Distribution of Items in the Pilot Test

Number Domain	Equal Groups	Multiplicative Comparison	Rectangular Area	Cartesian Product
Integers	3	3	3	3
Fractions	3	3	3	3

Each item was scored dichotomously. One point was awarded for a correct answer, and zero points were given for incorrect responses, including computational errors, clerical mistakes, incorrect equation setup, incorrect units, or missing units.

To establish validity, the test was reviewed by multiple experts in mathematics education and experienced primary school mathematics teachers through a triangulation process. For reliability, internal consistency was assessed using Cronbach's alpha coefficient with SPSS 20.0, yielding an alpha value of .852, which indicates good reliability.

Item analysis was conducted using data from the pilot test. Based on total test scores, the top eight students (approximately 28.5%) were classified as the high-achievement group (P_H), and the bottom eight students (approximately 28.5%) were classified as the low-achievement group (P_L). Item difficulty was calculated using the formula $P = (P_H + P_L)/2$, and item discrimination was calculated using the formula $D = P_H - P_L$. Table 2 presents the difficulty and discrimination indices for all pilot test items.

Table 2. Item Difficulty and Discrimination Indices for the Pilot Test

Item #	Percentage Correct (P_H)	Percentage Correct (P_L)	Item Diffi . (P)	Item Discri. (D)	Item #	Percentage Correct (P_H)	Percentage Correct (P_L)	Item Diffi . (P)	Item Discri. (D)
1	.88	.25	.56	.63	13	1	.38	.69	.63
2	.63	.13	.38	.50	14	1	.50	.75	.50
3	.75	0	.38	.75	15	.63	.38	.50	.25
4	1	.13	.56	.88	16	.88	.13	.50	.75
5	1	.13	.56	.88	17	1	0	.50	1

6	1	.25	.63	.75	18	.75	.38	.56	.38
7	.88	.38	.63	.50	19	.63	.75	.69	.13
8	.63	.50	.56	.13	20	.75	.13	.44	.63
9	.50	0	.25	.50	21	.88	.25	.56	.63
10	.75	.13	.44	.63	22	.88	.38	.63	.50
11	.75	.13	.44	.63	23	.50	.50	.50	0
12	.88	.63	.75	.25	24	1	.13	.56	.88

The analysis showed that all items had acceptable difficulty levels, ranging from .15 to .85. However, items 8, 12, 15, 19, and 23 had discrimination indices below .25 and were therefore removed from the final test. The finalised test items and their corresponding classifications are presented in Table 3.

Table 3. Mapping of Pilot Items to Final Test Items

Pilot Item No.	Pilot Item Type	Final Item No.
1	Multiplicative Comparison (Integer)	1
2	Area (Integer)	2
3	Equal Groups (Integer)	3
4	Cartesian Product (Integer)	4
5	Equal Groups (Fraction)	5
6	Area (Fraction)	6
7	Multiplicative Comparison (Fraction)	7
8	Cartesian Product (Fraction)	Deleted
9	Equal Groups (Integer)	8
10	Area (Integer)	9
11	Multiplicative Comparison (Integer)	10
12	Cartesian Product (Integer)	Deleted
13	Equal Groups (Fraction)	11
14	Multiplicative Comparison (Fraction)	12
15	Area (Fraction)	Deleted
16	Cartesian Product (Fraction)	13
17	Multiplicative Comparison (Integer)	14
18	Area (Integer)	15
19	Equal Groups (Integer)	Deleted
20	Cartesian Product (Integer)	16
21	Equal Groups (Fraction)	17
22	Area (Fraction)	18
23	Multiplicative Comparison (Fraction)	Deleted
24	Cartesian Product (Fraction)	19

To further illustrate the characteristics of the finalised test items, representative items with their problem types and discrimination indices are presented in Table 4.

Table 4. Representative Items with Problem Types and Discrimination Indices

Item No.	Problem Type	Number Domain	Item Description	Discrimination Index (D)
2	Rectangular Area	Integer	A classroom is 9 meters long and 8 meters wide. What is its area in square meters?	0.50

5	Equal Groups	Fraction	Producing one cup of orange juice requires $\frac{3}{8}$ of a bag of oranges. How many bags of oranges are needed to produce 24 cups of orange juice?	0.88
16	Cartesian Product	Integer	There are 4 boys and 6 girls dancing. How many boy–girl pairs can be formed?	1.00

Note. The table presents selected representative items from the final test to illustrate variations in problem types and discrimination indices.

The distribution of items in the final test across problem types and numerical contexts is shown in Table 5.

Table 5. Distribution of Items in the Final Test

Number Domain	Equal Groups	Multiplicative Comparison	Rectangular Area	Cartesian Product
Integers	2	3	3	2
Fractions	3	2	2	2

The final test consisted of 19 items and was administered in a paper-and-pencil format with a testing duration of 40 minutes. The test was conducted with 178 sixth-grade students from 15 rural primary schools in Taiwan. Because several pilot items were removed due to low discrimination indices, the final test did not contain an equal number of items in every structure-by-number-domain cell. To reduce the influence of unequal item counts, performance comparisons were based on average item-level accuracy rates rather than raw total scores.

3. FINDINGS AND DISCUSSION

Problem-Solving Performance in Integer and Fraction Multiplication Among Sixth-Grade Students in Rural Primary Schools in Taiwan

The maximum score of the test in this study was 19, and the average score was 12.39. The number of correct responses and accuracy rates for each item are presented in Table 6.

Table 6. Number of Correct Responses and Accuracy Rates for Each Item (N = 178)

Item No.	Number of Correct Responses	Accuracy Rate
1	132	.74
2	117	.66
3	126	.71
4	131	.74
5	132	.74
6	88	.49
7	96	.54
8	143	.80
9	148	.83
10	127	.71
11	107	.60
12	112	.63
13	87	.49
14	132	.74
15	127	.71
16	113	.63

17	107	.60
18	90	.51
19	91	.51

Based on Table 6, the average accuracy rates across different types of multiplication problems were further calculated, as shown in Table 7.

Table 7. Average Accuracy Rates Across Different Types of Multiplication Problems

Problem Type	Item Numbers	Average Accuracy Rate
Equal groups	3, 5, 8, 11, 17	0.69
Multiplicative comparison	1, 7, 10, 12, 14	0.67
Rectangular area	2, 6, 9, 15, 18	0.64
Cartesian product	4, 13, 16, 19	0.59

As shown in Tables 6 and 7, the overall accuracy rate of students in solving multiplication problems was approximately 65%, and 89% of the items had accuracy rates above 50%, suggesting that students demonstrated a basic level of competence in solving the multiplication problems included in this study.

Among the four types of multiplication problems, equal groups problems had the highest performance, with accuracy rates ranging from .60 to .80 and an average of .69. This was followed by multiplicative comparison problems (average = .67), area problems (average = .64), and Cartesian product problems (average = .59). Although differences among problem types were observed, all accuracy rates exceeded 50%, suggesting that students demonstrated a general level of competence across all multiplication structures.

Relationships in Problem-Solving Performance Across Four Multiplication Problem Types in Integer and Fraction Contexts Among Rural Sixth-Grade Primary Students in Taiwan

Equal Groups Problems

Five equal groups of items were included, consisting of two integer items and three fraction items. The accuracy rates are presented in Table 8.

Table 8. Accuracy Rates for Equal Groups Problems

Category	Item No.	Accuracy Rate	Average Accuracy
Integer	3	.71	.756
	8	.80	
Fraction	5	.74	.648
	11	.60	
	17	.60	

The average accuracy rate for integer items was .756, whereas that for fraction items was .648. This indicates that students performed better on integer problems than on fraction problems, with a difference of approximately 10.8%.

Multiplicative Comparison Problems

Five multiplicative comparison items were included, consisting of three integer items and two fraction items. The results are presented in Table 9.

Table 9. Accuracy Rates for Multiplicative Comparison Problems

Category	Item No.	Accuracy Rate	Average Accuracy
Integer	1	.74	.732
	10	.71	

	14	.74	
Fraction	7	.54	.584
	12	.63	

The average accuracy rate for integer items was .732, while that for fraction items was .584, showing a difference of approximately 14.8%. This suggests that students encountered greater difficulty when solving fraction problems within this structure.

Area Problems

Five area items were included, consisting of three integer items and two fraction items. The results are presented in Table 10.

Table 10. Accuracy Rates for Area Problems

Category	Item No.	Accuracy Rate	Average Accuracy
Integer	2	.66	.734
	9	.83	
	15	.71	
Fraction	6	.49	.50
	18	.51	

The average accuracy rate for integer items was .734, whereas that for fraction items was only .50. The difference of 23.4% represents the largest gap among all problem types, indicating that students experienced the greatest difficulty with fraction problems in area contexts.

Cartesian Product Problems

Four Cartesian product items were included, consisting of two integer items and two fraction items. The results are presented in Table 11.

Table 11. Accuracy Rates for Cartesian Product Problems

Category	Item No.	Accuracy Rate	Average Accuracy
Integer	4	.74	.685
	16	.63	
Fraction	13	.49	.50
	19	.51	

The average accuracy rate for integer items was .685, while that for fraction items was .50, indicating a difference of approximately 18.5%.

Taken together, the area and Cartesian product results show that both structures were challenging in fraction contexts, but the performance gap was larger for area problems (23.4%) than for Cartesian product problems (18.5%). One possible explanation is that both structures require coordination of two quantities, but they differ in the nature of that coordination. In area problems, students must coordinate two continuous measures, such as length and width, and interpret the product as a composite unit of area. In fraction contexts, this requires coordinating fractional lengths and understanding the resulting fractional area as a new quantity. In contrast, Cartesian product problems often involve discrete quantities and may allow students to rely more on concrete enumeration or pairing strategies. These patterns suggest that area problems may present particularly strong challenges when comparing integer and fraction multiplication contexts.

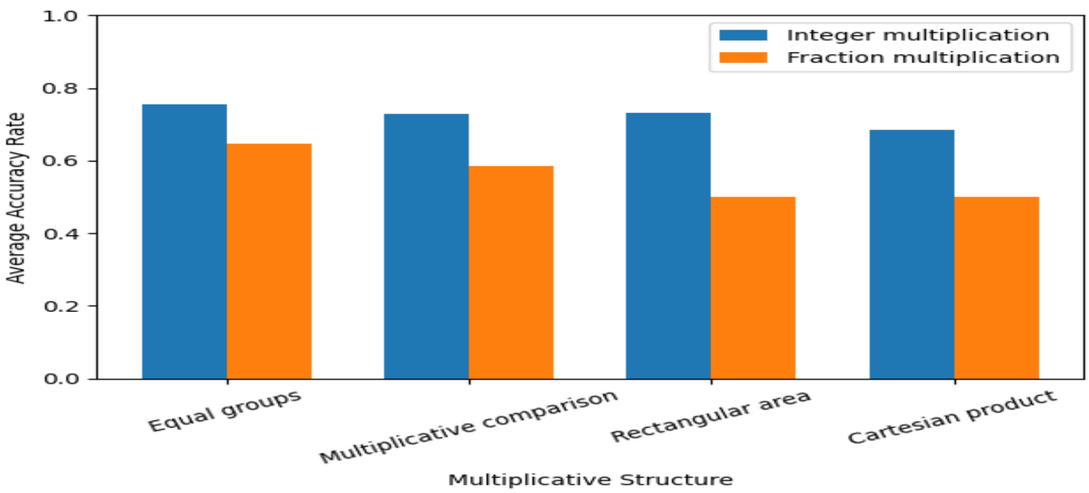

Figure 1. Average Accuracy Rates for Integer and Fraction Multiplication Across Four Multiplicative Structures

As shown in Figure 1, students consistently achieved higher accuracy in integer multiplication than in fraction multiplication across all four structures, with the magnitude of the differences varying by structure. The smallest difference was observed in equal groups problems, suggesting that students were relatively more successful in extending their understanding of integer multiplication to fraction contexts within this structure. In contrast, the largest difference was found in area problems, followed by Cartesian product and multiplicative comparison problems.

These findings indicate that students' performance in multiplication varies between integer and fraction contexts and is closely related to the underlying multiplicative structures. Students consistently demonstrated higher accuracy in integer problems than in fraction problems across all structures, indicating that fraction multiplication poses greater conceptual challenges. This result is consistent with previous research showing that fraction learning requires more complex conceptual understanding than whole-number operations, particularly in coordinating quantities and multiplicative relationships (Braithwaite & Siegler, 2021).

Furthermore, the variation in performance across problem types suggests that different multiplicative structures may impose different cognitive demands. This finding is consistent with Greer's (1992) argument that multiplication problems differ in their underlying quantitative relationships and therefore may require different forms of reasoning. The present results further suggest that these structural differences become more pronounced when students solve fraction multiplication problems. The relatively smaller gap observed in equal groups problems may be attributed to their closer alignment with repeated addition, allowing students to extend their existing understanding more easily. In contrast, larger gaps in area and Cartesian product problems indicate that these structures require more complex reasoning and the integration of multiple representations. This interpretation is supported by research showing that students' understanding of fractions is influenced by how they interpret numerical representations and relationships among quantities (Liu & Braithwaite, 2023). In addition, students' performance in fraction-related tasks is closely associated with their level of conceptual understanding and fraction sense (Jordan et al., 2024).

Overall, these results suggest that students' performance in fraction multiplication varies across different multiplicative structures and that some structures appear to present greater challenges than others. This perspective is further supported by research indicating that fraction learning involves complex interactions among conceptual knowledge, misconceptions, and cognitive development (D'Erchie et al., 2026; Xu et al., 2024).

4. CONCLUSION

This study investigated the multiplication problem-solving performance of sixth-grade students in rural primary schools in Taiwan and examined differences across various multiplicative structures in integer and fraction contexts. The results indicate that students generally performed better in integer multiplication than in fraction multiplication. Across different problem types, equal groups problems yielded the highest accuracy rates, followed by multiplicative comparison, area, and Cartesian product problems. Overall, all accuracy rates were above 50%, suggesting that students demonstrated a basic level of competence in solving multiplication problems; however, variations across problem types indicate differences in the depth of conceptual understanding across multiplicative structures.

Further analysis revealed that the gap between integer and fraction performance was not consistent across problem types. The largest difference was observed in area problems, followed by Cartesian product and multiplicative comparison problems, whereas the smallest difference was found in equal groups problems. These findings suggest that students' difficulties in fraction multiplication may vary across multiplicative structures, particularly in tasks involving the coordination of multiple relationships and higher cognitive demands.

Consistent with prior research on fraction learning, students in this study performed better in integer multiplication than in fraction multiplication and experienced greater difficulty in problem types requiring the integration of multiple representations and relationships (Liu & Braithwaite, 2023). Within the rural sixth-grade sample examined in this study, area and Cartesian product problems appeared to present greater challenges than equal groups and multiplicative comparison problems.

Based on these findings, instruction may place greater emphasis on conceptual understanding in fraction multiplication rather than focusing solely on procedural computation. Teachers may provide structure-sensitive instruction for different multiplicative situations. For area problems, concrete area models and visual representations may help students connect fractional lengths with fractional areas before introducing abstract formulas. For Cartesian product problems, systematic listing strategies, pairing activities, and array representations may support students in coordinating two quantities. For equal groups problems, instruction may build on students' existing understanding of repeated addition to facilitate connections between integer and fraction multiplication contexts. Instructional support should also address the conceptual demands associated with each structure, such as coordinating fractional units in area problems and distinguishing multiplicative relationships from additive reasoning in comparison problems.

To support students in rural schools, educational authorities may provide teachers with instructional resources and professional development opportunities focused on structure-specific approaches to multiplication. Such support may help teachers use visual models, contextualised tasks, and multiple representations when teaching different multiplicative structures.

Several limitations should be acknowledged. First, the pilot study involved a relatively small sample (N = 28) and was intended primarily for preliminary item screening; therefore, the item statistics should be interpreted with appropriate caution. Second, the present study focused on students' accuracy rates and did not analyse error types or solution strategies. Future research may examine conceptual, procedural, and representation-related errors across different multiplicative structures. Third, because the study did not include urban or national comparison groups, the findings should be interpreted as descriptive of rural sixth-grade students rather than as evidence of rural–urban differences.

Overall, this study contributes to the literature on multiplicative reasoning by showing that performance patterns vary across multiplicative structures when students solve integer and fraction multiplication problems. The findings suggest that the cognitive demands associated with different multiplicative structures may contribute to students' difficulties in fraction multiplication and provide a more nuanced understanding of variation in multiplication problem-solving performance across contexts.

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